STUDENT NUMBER: 
NAME-SURNAME: 
SIGNATURE: 
INSTRUCTOR: Emre Sermutlu 
DURATION: 90 minutes

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IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.
2) Check that the exam paper contains 5 problems.
3) Show all your work. No points will be given to correct answers without reasonable work.
Capacitor: $q = CV$

Capacitance of Parallel Plate Capacitor: $C = \frac{\varepsilon_0 A}{d}$

Capacitors in Parallel: $C_{eq} = \sum_{i=1}^{n} C_i$

Capacitors in Series: $\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$

Energy Stored in a Capacitor: $U = \frac{1}{2} CV^2 = \frac{Q^2}{2C}$

Energy Density: $u = \frac{1}{2} \varepsilon_0 E^2$

Effect of Dielectric: $C = \kappa C_{air}$

Resistance of a Conductor: $R = \frac{V}{i}$

Resistance of a Wire of Length $L$, Cross-section $A$: $R = \rho \frac{L}{A}$

Power in Electrical Device: $P = iV$

Series Resistances: $R_{eq} = \sum_{j=1}^{n} R_j$

Parallel Resistances: $\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$

Magnetic Force on a Moving Particle: $\vec{F}_B = q \vec{v} \times \vec{B}$

Charged Particle Circulating in a Magnetic Field: $r = \frac{mv}{|q|B}$

Magnetic Force on a Current - Carrying Wire: $\vec{F}_B = i \vec{L} \times \vec{B}$

Biot - Savart Law: $d\vec{B} = \frac{\mu_0 i \, ds \times \hat{r}}{4\pi r^2}$

Magnetic Field of a Long Straight Wire: $B = \frac{\mu_0 i}{2\pi R}$

Magnetic Field of a Circular Arc (at center): $B = \frac{\mu_0 i \phi}{4\pi R}$

Force Between Parallel Currents: $F_{ba} = \frac{\mu_0 L_i a i_b}{2\pi d}$

Ampere’s Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$
1) a) A capacitor stores an energy of $1\mu j$ when the potential difference between its plates is $3\, V$. Find its capacitance.

b) Find the charge on the $4\, pF$ capacitor in the figure.

---

**Diagram**

![Diagram showing capacitors and 60V applied voltage](image)

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**Answer:**

a)

\[
U = \frac{1}{2} CV^2
\]

\[
C = \frac{2U}{V^2} = \frac{12\mu j}{(3\, V)^2}
\]

\[
C = 0.22\, pF = 2.2 \times 10^{-7}\, F
\]

b)

\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1\, pF} + \frac{1}{4\, pF}
\]

\[
C_{eq} = 0.8\, pF
\]

\[
q = VC = 60\, V \times 0.8\, pF = 48\, pC
\]

\[
q_1 = q_2 = 4.8 \times 10^{-11}\, C
\]
2) A cylindrical wire made of Nichrome has radius \( r = 1 \text{ mm} \) and length \( L \). Nichrome’s resistivity is \( \rho = 5 \times 10^{-7} \Omega \cdot m \). When a potential difference of 100 \( V \) is applied, the power dissipation is 2000 \( W \). Find \( L \).

Answer:

\[
P = \frac{V^2}{R} = \frac{(100V)^2}{2000W} = 5 \Omega
\]

\[
R = \frac{\rho L}{A} \Rightarrow L = \frac{AR}{\rho}
\]

\[
L = \frac{\pi (10^{-3}m)^2 5\Omega}{5 \times 10^{-7} \Omega \cdot m} = 10\pi = 31.4 \text{ m}
\]
3) In the following circuit,
   a) Find all currents
   b) Find \( V_{ab} \).

Answer:

\[
\begin{align*}
80 - 7(i_1 + i_2) - i_1 - 7 &= 0 \\
7 + i_1 - 10i_2 + 60 &= 0
\end{align*}
\]

\[
\begin{align*}
8i_1 + 7i_2 &= 73 \\
-i_1 + 10i_2 &= 67
\end{align*}
\]

The solution of the system of equations give:

\[
\begin{align*}
i_2 &= 7 A, \quad i_1 = 3 A, \quad i_1 + i_2 = 10 A \\
V_{ab} &= +7 + 3 - 56 = -46 V
\end{align*}
\]
4) A charged particle with mass \( m \), charge \( q \), kinetic energy \( K \) enters a magnetic field and then follows a circle of radius \( r \) as seen in the figure. Find the magnitude and direction of the magnetic field \( \vec{B} \).

\[ K = \frac{1}{2}mv^2 \ \Rightarrow \ v = \sqrt{\frac{2K}{m}} \]

\[ r = \frac{mv}{qB} \]

\[ B = \frac{mv}{qr} = \frac{m\sqrt{\frac{2K}{m}}}{qr} \]

\[ B = \frac{\sqrt{2Km}}{qr} \]

Using the right hand rule, we see that \( \vec{B} \) is into the page.
5) A current of $I$ flows around a square of side length $a$. Find the magnitude of the magnetic field at the center of the square.

Answer:

Using Biot-Savart law for one side of the square, we obtain the magnetic field at $C$ as:

$$B = \int_{-\pi/4}^{\pi/4} \frac{\mu_0 I \cos \theta d\theta}{4\pi (a/2)}$$

$$B = \frac{\mu_0 I}{2\pi a} \sin \theta \bigg|_{-\pi/4}^{\pi/4}$$

$$B = \frac{\mu_0 I \sqrt{2}}{2\pi a}$$

If we add the magnetic fields of all four sides:

$$B_{total} = 2\sqrt{2} \frac{\mu_0 I}{\pi a}$$