ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

**PHYS 122 - General Physics II**

# FIRST MIDTERM EXAMINATION
17.03.2010

**STUDENT NUMBER:**

**NAME-SURNAME:**

**SIGNATURE:**

**INSTRUCTOR:** Emre Sermutlu

**DURATION:** 90 minutes

| Question | Grade | Out of |
|---|---|---|
| 1 | | 20 |
| 2 | | 20 |
| 3 | | 20 |
| 4 | | 20 |
| 5 | | 20 |
| Total | | 100 |

**IMPORTANT NOTES:**

**1)** Please make sure that you have written your student number and name above.

**2)** Check that the exam paper contains 5 problems.

**3)** Show all your work. No points will be given to correct answers without reasonable work.

Coulomb's Law: $F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1|\,|q_2|}{r^2}$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2, \qquad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N.m^2/C^2$

Elementary Charge: $e = 1.6 \times 10^{-19}\ C$

Electric Field: $\overrightarrow{E} = \frac{\overrightarrow{F}}{q_0}$

Electric Field of a Point Charge: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Electric Field of a Dipole: $E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}, \quad p = qd$

Electric Field of a Charged Ring: $E = \frac{1}{4\pi\varepsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}$

Electric Field of a Charged Disk: $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$

Force on a Point Charge in an Electric Field: $\overrightarrow{F} = q\overrightarrow{E}$

Electric Flux: $\Phi = \oint \overrightarrow{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

Infinite Line of Charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Infinite Nonconducting Plane: $E = \frac{\sigma}{2\varepsilon_0}$

Spherical Shell of Charge: $E_{in} = 0, \quad E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Finding $V$ from $\overrightarrow{E}$: $V_f - V_i = -\int_i^f \overrightarrow{E} \cdot d\vec{s}$

Potential of a Point Charge: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Potential of Several Point Charges: $V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$

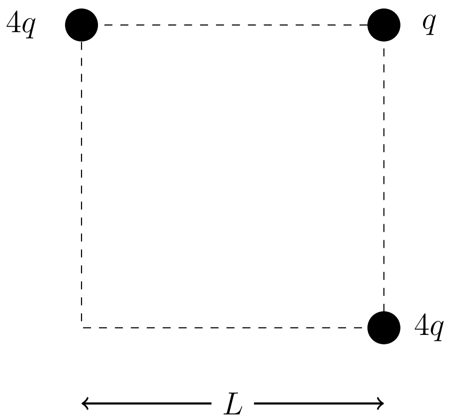
Potential of a Continuous Charge Distribution: $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$

Potential of a Charged Disk: $V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$

Finding $\overrightarrow{E}$ from $V$: $E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$

Electric Potential Energy of Two Charges: $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

**1)** Three charges are fixed at the corners of a square of side length $L$. We want the net force on charge $q$ to be zero. What is the charge we have to put on the fourth corner?




**Answer:**
Let's call the fourth charge $Q$. The force between $q$ and $Q$ is:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qq}{(L\sqrt{2})^2}\left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right)\right] = 0$$

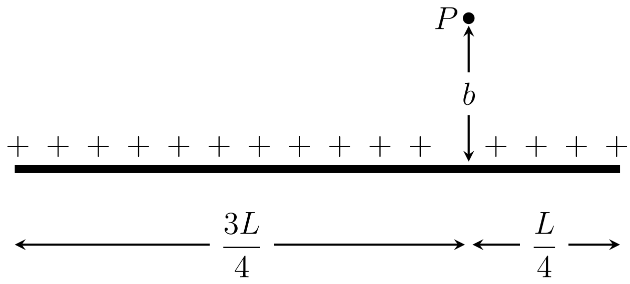
The net force on $q$ must be zero:

$$\vec{F}_{net} = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{L^2}\vec{i} + \frac{4q^2}{L^2}\vec{j} + \frac{Qq}{2L^2}\left(\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right)\right] = 0$$

$$\frac{4q^2}{L^2} + \frac{\sqrt{2}Qq}{4L^2} = 0$$

$$Q = -8\sqrt{2}q$$

**2)** A total charge $Q$ is distributed uniformly on a rod of length $L$. Find the electric field at the point $P$ in unit vector notation. (Do not evaluate the integrals)




**Answer:**

$$dq = \frac{Q}{L}\,dx$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{b^2 + \left(\frac{3L}{4} - x\right)^2}$$

$$dE_x = dE\sin\theta, \qquad dE_y = dE\cos\theta$$

$$E_x = \frac{Q}{4\pi\varepsilon_0 L}\int_0^L \frac{\frac{3L}{4} - x}{\left[b^2 + \left(\frac{3L}{4} - x\right)^2\right]^{3/2}}\,dx$$

$$E_y = \frac{Q}{4\pi\varepsilon_0 L}\int_0^L \frac{b}{\left[b^2 + \left(\frac{3L}{4} - x\right)^2\right]^{3/2}}\,dx$$

or alternatively

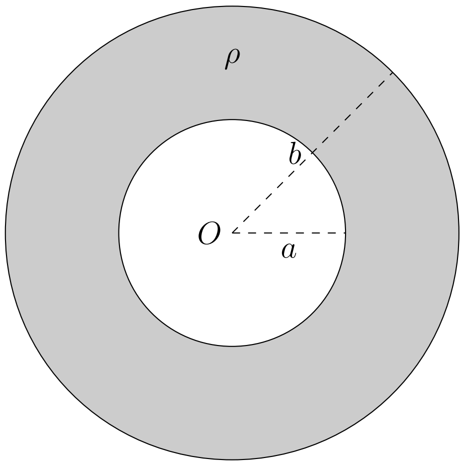
$$E_x = \frac{Q}{4\pi\varepsilon_0 L}\int_{-\frac{3L}{4}}^{\frac{L}{4}} \frac{x}{\left[b^2 + x^2\right]^{3/2}}\,dx$$

$$E_y = \frac{Q}{4\pi\varepsilon_0 L}\int_{-\frac{3L}{4}}^{\frac{L}{4}} \frac{b}{\left[b^2 + x^2\right]^{3/2}}\,dx$$

$$\overrightarrow{E} = E_x\,\overrightarrow{i} + E_y\,\overrightarrow{j}$$

**3)** A non-conducting spherical shell has inner radius $a$ and outer radius $b$. It has uniform volume charge density $\rho$. Find the electric field for

**a)** $r < a$
**b)** $a < r < b$
**c)** $b < r$




**Answer:**
Using Gauss' Law:

**a)** $$4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0}$$

$$E = 0$$

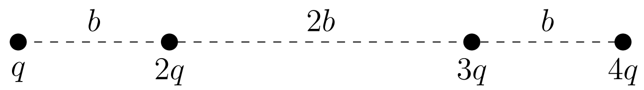
**b)** $$4\pi r^2 E = \frac{\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)\rho}{\varepsilon_0}$$

$$E = \frac{(r^3 - a^3)\rho}{3r^2\varepsilon_0}$$

**c)** $$4\pi r^2 E = \frac{\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)\rho}{\varepsilon_0}$$

$$E = \frac{(b^3 - a^3)\rho}{3r^2\varepsilon_0}$$

**4)** Find the work we have to do to assemble the system shown in figure, bringing each charge from infinity:




**Answer:**

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q\,2q}{b} + \frac{q\,3q}{3b} + \frac{q\,4q}{4b} + \frac{2q\,3q}{2b} + \frac{2q\,4q}{3b} + \frac{3q\,4q}{b}\right)$$

$$U = \frac{65}{3}\,\frac{q^2}{4\pi\varepsilon_0 b}$$

**5)** On a certain region of space, the electric potential is given by $V(x, y, z) = \sqrt{x^2 + y^2} - xyz^2$. Find the electric field.

**Answer:**

$$\overrightarrow{E} = -\frac{\partial V}{\partial x}\overrightarrow{i} - \frac{\partial V}{\partial y}\overrightarrow{j} - \frac{\partial V}{\partial z}\overrightarrow{k}$$

$$\overrightarrow{E} = \left(yz^2 - \frac{x}{\sqrt{x^2 + y^2}}\right)\overrightarrow{i} + \left(xz^2 - \frac{y}{\sqrt{x^2 + y^2}}\right)\overrightarrow{j} + 2xyz\,\overrightarrow{k}$$