ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

PHYS 122 - General Physics II

FINAL EXAMINATION
08.06.2010

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Emre Sermutlu

DURATION: 90 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N.m^2, \qquad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N.m^2/C^2$

Capacitor: $q = CV$, Parallel Plate Capacitor: $C = \frac{\varepsilon_0 A}{d}$

Energy Stored in a Capacitor: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Effect of Dielectric : $C = \kappa C_{air}$

Power in Electrical Device: $P = iV$

Charging a Capacitor: $q = C\mathcal{E}(1 - e^{-t/RC})$

Discharging a Capacitor: $q = q_0 e^{-t/RC}$

Magnetic Force on a Moving Particle: $\vec{F}_B = q\overrightarrow{v} \times \overrightarrow{B}$

Charged Particle Circulating in a Magnetic Field: $r = \frac{mv}{|q|B}$

Permeability Constant: $\mu_0 = 4\pi \times 10^{-7} Tm/A$

Magnetic Field of a Long Straight Wire: $B = \frac{\mu_0 i}{2\pi R}$

Magnetic Field of a Circular Arc (at center): $B = \frac{\mu_0 i \phi}{4\pi R}$

Force Between Parallel Currents: $F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}$

Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0\, I_{\text{enc}}$

Faraday's Law: $\mathcal{E} = -N\frac{d\Phi_B}{dt}$

Magnetic Energy of Inductor: $U = -\frac{1}{2}Li^2$

Impedance: $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Phase constant: $\tan\phi = \frac{X_L - X_C}{R}$

Power in RLC circuit: $P_{avg} = \mathcal{E}_{rms} I_{rms} \cos\phi$

Capacitive Reactance: $\frac{1}{\omega C}$, Inductive Reactance: ωL

Ampere - Maxwell Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0\, I_{\text{enc}}$

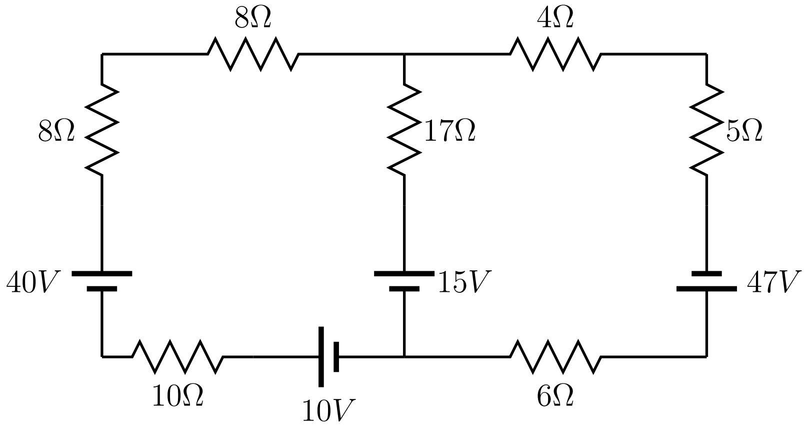
Displacement Current: $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

Plane Electromagnetic Wave: $\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

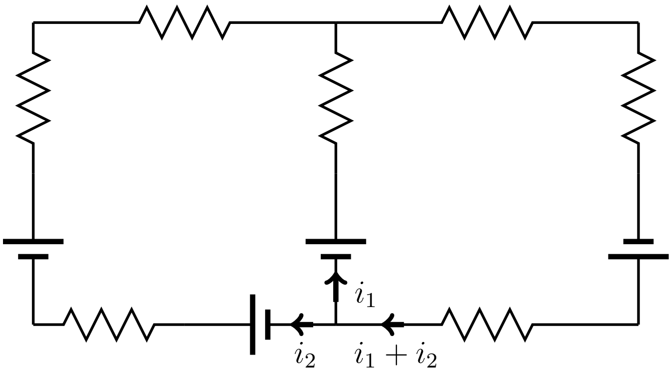
Poynting Vector: $\overrightarrow{S} = \frac{1}{\mu_0}\overrightarrow{E} \times \overrightarrow{B}$

Intensity: $I = \frac{1}{c\mu_0} E^2_{rms}$

1) Find all currents in the following circuit:



Answer: Let's choose the currents as follows:



Now, using Kirchoff's loop rule, we obtain:

$$+10 - 10i_2 + 40 - 8i_2 - 8i_2 + 17i_1 - 15 = 0$$

$$+15 - 17i_1 - 4(i_1 + i_2) - 5(i_1 + i_2) + 47 - 6(i_1 + i_2) = 0$$

$$-17i_1 + 26i_2 = 35$$

$$32i_1 + 15i_2 = 62$$

The solution of this system of equations give:

$$i_1 = 1A, \quad i_2 = 2A, \quad i_1 + i_2 = 3A$$

2) a) The time constant of an RC circuit is $\tau = RC = 0.05\,s$. Find the time necessary for initially uncharged capacitor to be charged to 99% of its final charge.

Answer:

$$q = C\mathcal{E}(1 - e^{-t/RC})$$

Final charge is $C\mathcal{E}$ therefore $q = 0.99\,C\mathcal{E}$.

$$0.99 = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = 0.01$$

$$-t/\tau = \ln 0.01$$

$$t = 0.05s. \times 4.6 = 0.23s.$$

b) A particle of mass m, charge q travels in a circular path of radius R in a uniform magnetic field B. Find its period of revolution.

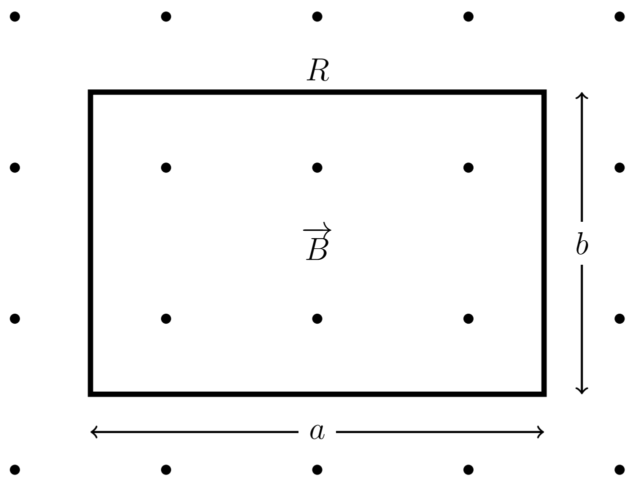
Answer:

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi m}{qB}$$

3) A variable magnetic field of magnitude $B = B_0(1 + t^3)$ is perpendicular to a rectangular loop. The loop has dimensions $a \times b$ and resistance R. Find the magnitude and direction of the induced current.



Answer:

$$\Phi_B = BA = Bab = B_0(1 + t^3)ab$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B_0 3t^2 ab$$

$$I = \frac{\mathcal{E}}{R} = -\frac{3B_0 t^2 ab}{R}$$

Positive direction is counter-clockwise.
Therefore I is in clockwise direction.

4) a) In a series RLC circuit, $L = 90\,mH$, $C = 1.5\,\mu F$ and $f = 0.8\,kHz$. If the phase constant is 55°, what is the resistance of the coil?

Answer:

$$\omega = 2\pi f = 5027 s^{-1}$$

$$X_L = \omega L = 452\,\Omega$$

$$X_C = \frac{1}{\omega C} = 132\,\Omega$$

$$\frac{X_L - X_C}{R} = \tan 55 = 1.428$$

$$R = \frac{X_L - X_C}{\tan\phi} = 224\,\Omega$$

b) An oscillating LC circuit is made of a $4\,nF$ capacitor and a $9\,mH$ inductor. The maximum current is $3\,A$. What is the maximum charge on the capacitor?

$$\frac{1}{2} L I_{max}^2 = \frac{1}{2}\frac{q_{max}^2}{C}$$

$$q_{max} = \sqrt{LC}\, I$$

$$q_{max} = \sqrt{9 \cdot 10^{-3} \cdot 4 \cdot 10^{-9}}\; 3$$

$$q_{max} = 18\,\mu C$$

5) **a)** The displacement current through a rectangular loop of area $3.5\,m^2$ is $8\,A$. At what rate is the electric field through the loop changing?

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{8}{8.85 \cdot 10^{-12} \cdot 3.5}$$

$$\frac{dE}{dt} = 2.58 \times 10^{11} V/m.s$$

b) The maximum electric field of an electromagnetic wave is $5\,V/m$. What is the intensity?

$$I = \frac{E_{rms}^2}{c\mu_0} = \frac{E_m^2}{2c\mu_0}$$

$$I = \frac{5^2}{2 \cdot 3 \cdot 10^8 \cdot 4\pi \cdot 10^{-7}}$$

$$I = 0.033\,W/m^2$$